# Question #4

## Implementing Linearized Kalman Filter (LKF)

To implement the LKF for this Cooperative Localization problem we define an LKF function which performs the prediction [Equation ( 1 )] and correction [Equation ( 2 )] step.

The function takes in:

* the ground truth values (state dynamics and measurements),
* the nominal state and measurement trajectories without process or measurement noise,
* the process noise covariance () for tuning and,
* initial values for and .

The function outputs:

* the state estimates (),
* measurement estimates ,
* estimation errors ( and ) and,
* sequence of covariance matrices for state and measurement estimates ( and)

The LKF runs the following set of equations in a loop for steps:

|  |  |  |
| --- | --- | --- |
|  |  | ( ) |

|  |  |  |
| --- | --- | --- |
|  |  | ( ) |

This function is included in a Monte Carlo run which feeds the ground truth models, nominal state and measurement trajectories, and the DT state space matrices linearized around the nominal trajectories () according to the methods from Question #2 and Question #3.

## Truth Model Testing (TMT) for LKF

For the TMT we do 50 Monte Carlo () runs. This is an appropriate number of Monte Carlo runs as we will have enough data sets for performing an unbiased NEES and NIS test.

The truth model is a simulated run of the nonlinear model with the process and measurement noise generated using the covariances uploaded on Canvas. We seed for the ground truth values for every Monte Carlo run from and then and then for every subsequent values the process noise is obtained from covariance matrix ‘Qtrue’.

Additionally, for the multiple runs in the TMT we feed the LKF the following initial values:

|  |  |  |
| --- | --- | --- |
|  |  | ( ) |

These initial conditions were selected for the following reasons: we are given the nominal and truth state trajectories, and hence the initial perturbation can be derived from those values; furthermore, we have a sufficient degree of certainty of our initial state perturbation such that the state perturbation covariance is finite (). And also, the angles cannot exceed 180o or go below -180o in radians.

In addition to this, in the LKF, the covariance matrix for the measurement noise () is set to be the one uploaded on canvas, i.e. Rtrue, as that information is generally known to us from the sensors being used.

## Tuning

There are two aspects that we need to keep in mind while tuning the Kalman filter. First, we need to make sure that the Kalman filter ‘works properly’, i.e. the estimate error averages at 0, and the bounds are converging. Second, we also look at the NEES and NIS chi-square tests and make sure they are within the confidence intervals.

We calculate the confidence intervals on MATLAB with significance level . We chose this significance level to provide a less stringent condition for proving whether the LKF is doing its job (i.e. having a low enough false-alarm probability). This was because we found that the LKF is not good enough for estimating this nonlinear system. The bounds are calculated using the MATLAB function ‘chi2inv’. As a result, for Monte Carlo runs, states, measured values, the chi-square confidence bounds are:

|  |  |  |
| --- | --- | --- |
|  |  | ( ) |

The and values are obtained by averaging the and values are all steps, which are obtained from the following equations:

While tuning the LKF we vary our predicted process noise covariance matrix () until the two conditions in the beginning of this subsection are satisfied. We first start with and run the truth model tests and check if the KF works (by looking at the state and measurement estimate errors) and the NEES and NIS tests are satisfied. The result showed that neither of the conditions were met, hence we moved to then to and then fine-tuned the main diagonal parameters in this way until the conditions were sufficiently satisfied.

In tuning, we focused on the consistency of the estimates for the and states as they appeared to be the states most sensitive to changes in .

In the end, the process noise covariance matrix for the LKF that best satisfied the conditions was found to be:

|  |  |  |
| --- | --- | --- |
|  |  | ( ) |

Before we move forward, the ‘best satisfying ’ is used lightly because, over multiple trials, it was found that the LKF cannot successfully estimate the states for this problem even after multiple variations. It is predicted that more fine tuning is required, however with 6 states to work with and limited time, we think that it is better to look for alternate options.

Referring to Figure 4 and Figure 5, we see that the LKF has done a decent job in estimating the measurements however, every time the value peaks, there is a large estimation error. This error could not be reduced regardless of the corresponding elements selected, and this resulted in the estimation errors carrying over. For instance we and also have large estimation errors at the same time points ( and ). We know from the linearization strategy in Question #1 and Question #2, that the UGV’s states are observed by feeding the UAV’s sensed values through trigonometric equations. And because of the large estimation errors in , the UGV’s states are adversely affected.

Referring now to Figure 2 and Figure 3. At time ~25s, where the first large deviation in occurs, the estimates for and shift greatly from the ‘correct path’ while still maintaining the ‘correct pattern’. This means, that the shape of the estimate state curve, matches the ground truth curve, however the estimation deviates from the ground truth. After that, the states and start to recover, however at ~75s again there is a large deviation in from the sensor which causes the states and to shift again. Essentially, because of the error in the sensor estimates for and , the state estimates for the UGV are being affected. The cause for the large deviation will be hypothesized and discussed in the next subsection.

Although, the angle estimates and are not significantly affected as they are more dependent on the control inputs. However we do see intermitted deviations at the 25s and 75s marks in the state estimates for in Figure 3.

Now while the state estimation errors for and do not converge, their covariances do. In fact, all the covariances of the state estimates () and measurement estimates () converge.

We will not look at the NEES and NIS tests for the LKF. Referring to Figure 1, we can see that the NIS test is, too and extent, being satisfied, however at the 25s and 75s marks the points go beyond the and limits. This overlaps with the previous observation of how the measurement estimates for deviate significantly at those times.

We can also see how the effect of this deviation carries over to the state estimate errors by looking at the NEES test where the points are within the confidence interval in the beginning but very quickly they go beyond the limit.

This tells us that the current configuration for the LKF fails the NEES and NIS tests. To reiterate, there is still room for more fine tuning of the LKF, but we believe that rather than wasting more time in tuning this further, a better alterative would be preferred.

## Comment on LKF for Estimating Nonlinear System

The tuning procedure was arduous and complicated. However, after multiple attempts, we can conclude that the LKF is not a good enough filter to estimate the nonlinear system for cooperative localization.

We derived this conclusion because we were not able to tune the LKF to satisfy the two conditions: the LKF ‘works properly’ and satisfies the NEES and NIS chi-square tests.

Referring to the UAV’s states, it seems that the LKF starts to lose accuracy in its states when the aircraft is moving near the boundaries of its domain in and . After some analysis, we believe that this is because of the large covariance for the sensed values , and . The latter two directly correspond to the UAV’s state while contains the UAV’s states and UGV’s states combined, the resulting ground truth measurement values (with the synthesized measurement noise) are believed to deviate beyond the bounds of the nominal trajectory, especially when the UAV is near the edge of its domain.

As a result, the LKF is not able to correctly estimate the measurement perturbations at the time 25s and 75s when the UAV is at the farthest point, which carries over to the corrected state perturbation estimate .

We believe, a better sensor could improve the state estimates obtained from the LKF. However, the issue of deviating beyond the nominal trajectory can be avoided by using the EKF which calculates the nominal trajectory regularly from the nonlinear model.

We also tested the filter by checking whether angle wrapping between operations (for , , and ) effected or improved the results (i.e. wrapping the nominal and ground truths before feeding into LKF). The result was worse off, because the filter would fail to estimate the states at the point where the angle wrapped back to within the intervals. Hence angles are not wrapped until after the estimate is obtained through the filter.

The MATLAB codes for the LKF implementation, the Monte Carlo runs, and the NEES and NIS tests are attached in Appendix D.

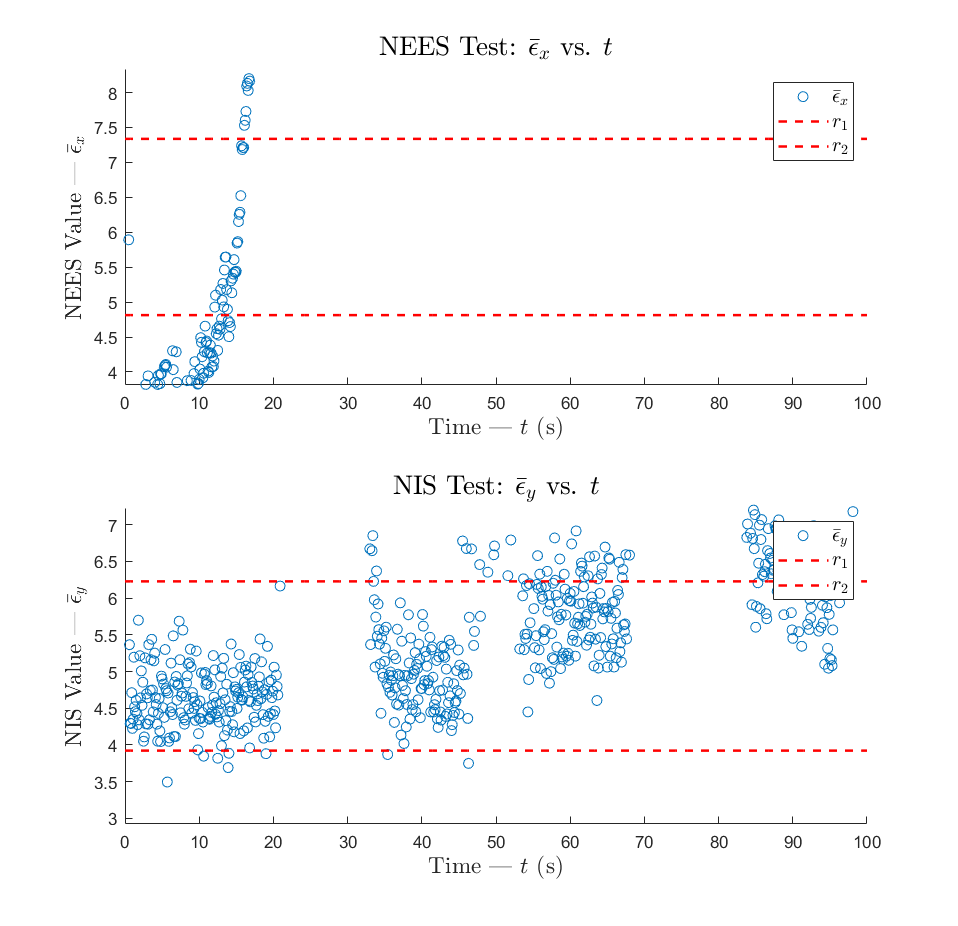


Figure – NEES and NIS chi-square test for LKF Monte Carlo runs with Equations ( 3 )( 4 )( 5 ) applied

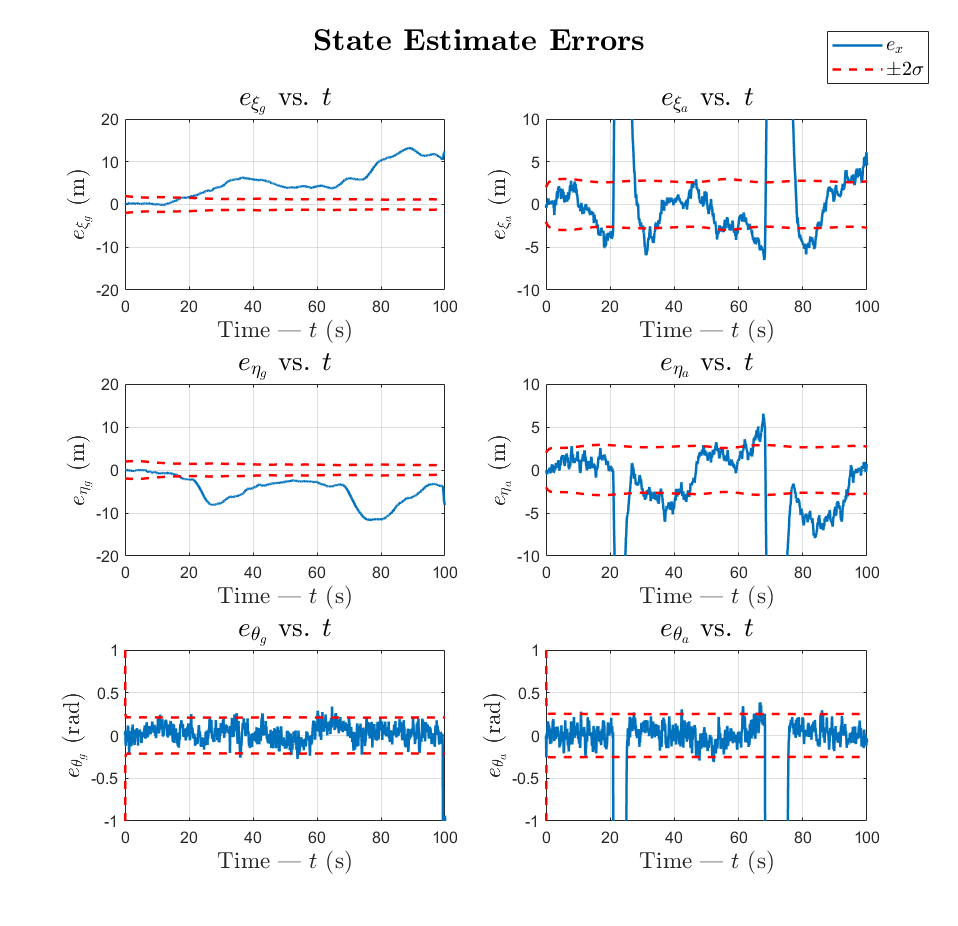


Figure – State estimate errors and bounds of sample LKF Monte Carlo run with Equations ( 3 )( 4 )( 5 ) applied

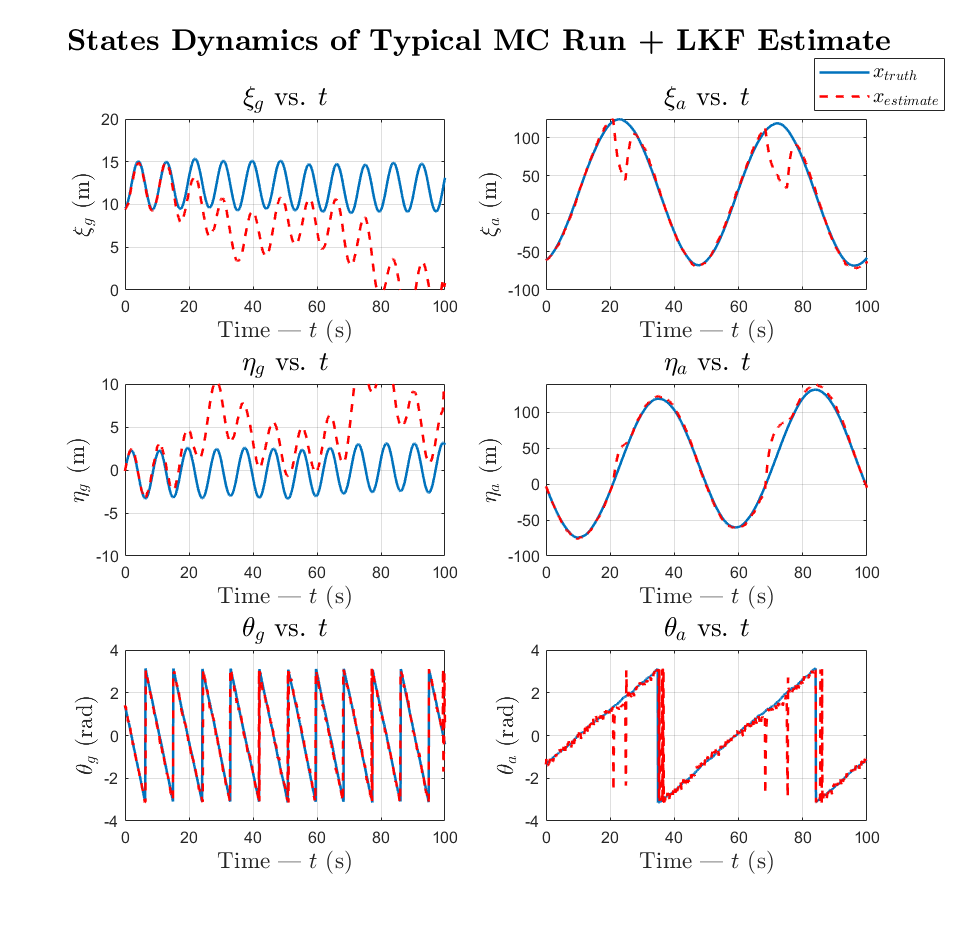


Figure – State estimates and state ground truth of sample LKF Monte Carlo run with Equations ( 3 )( 4 )( 5 ) applied

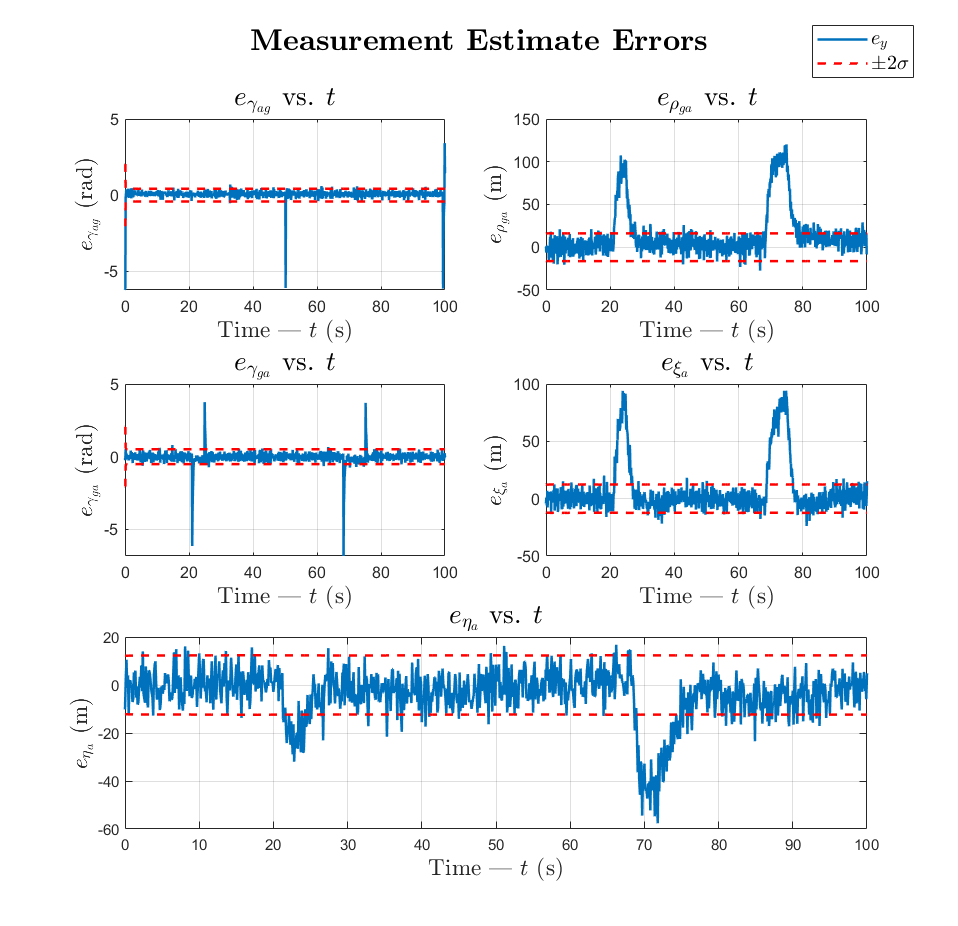


Figure – Measurement estimates and ground truth of sample LKF Monte Carlo run with Equations ( 3 )( 4 )( 5 ) applied

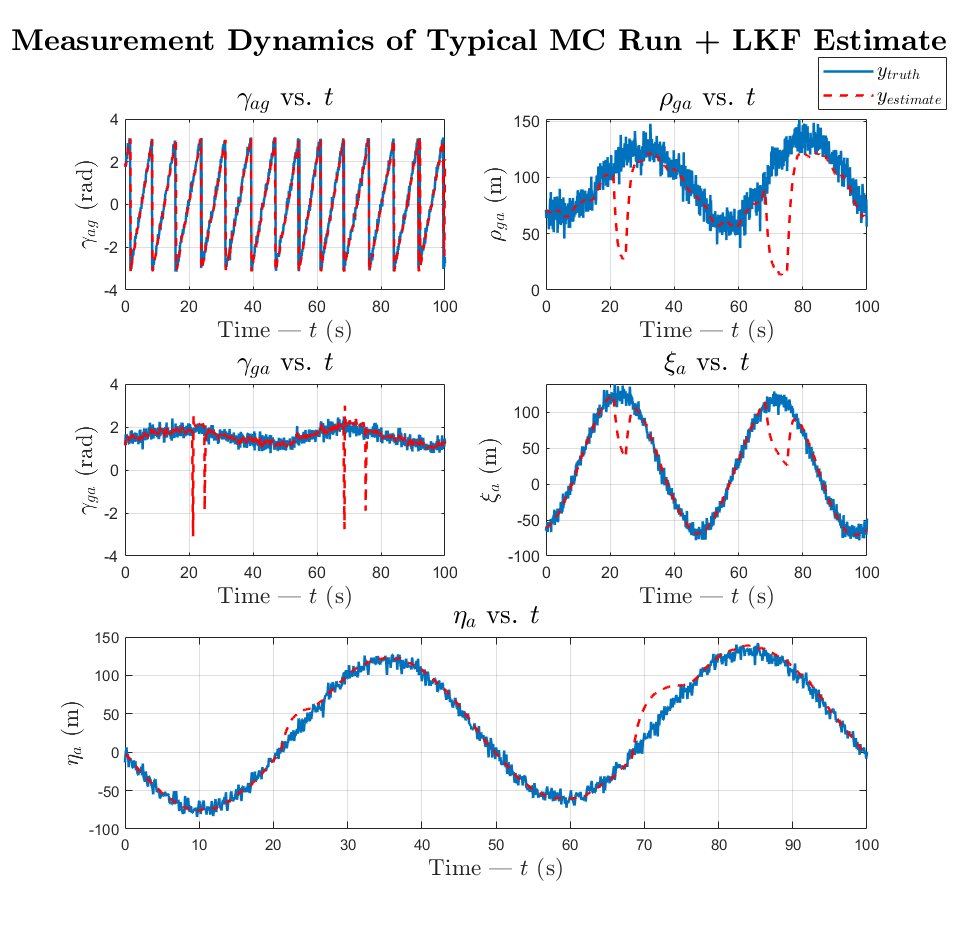


Figure – Measurement estimates and ground truth of sample LKF Monte Carlo run with Equations ( 3 )( 4 )( 5 ) applied